$\boxed{\text{Objective:}}$

To produce a given quantity as inexpensively as possible.

Constrained optimization

Production Function

$$Q = f(K, L) = K^{.5} L^{.5}$$

capital labor

Constrained Optimization

Recall: Our goal is to produce a given quantity (Q^*) for the lowest cost.

$$\min \text{ Cost}$$
$$\text{s.t. } Q = Q^*$$

$$\min \ RK + WL$$
$$\text{s.t. } f(K,L) = Q^*$$

How can we solve this?
What should be true?

ex: say the solution is K^*, L^*.

what must be true?

Ex: $Q^* = 10$ $\quad W = 2 \quad R = 4$

$L=0, K=0, Q=0$ $\quad MP_L = 4 \quad MP_K = 2$
$C=0$

$L=1 \; K=0, Q=4$ $\quad MP_L = 2 \quad MP_K = 3$
$C=2$

$L=2 \; K=0 \; Q=6$ $\quad MP_L = 1 \quad MP_K = 4$
$C=4$

$L=2 \; K=1 \; \underline{Q=10}$
$C=8$

Here we choose the max between $\frac{MP_L}{w}$ and $\frac{MP_K}{r}$

- We want to maximize marginal product per cost
  - note: cost (W, R) are constant, if marginal product is decreasing, then $\frac{MP}{Cost}$ decreases. This usually leads to an interior solution

- If we have an interior solution,

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y} \rightarrow \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

$$\boxed{\frac{MP_{L^*}}{W} = \frac{MP_{K^*}}{R}}$$

- Our solution should, for all inputs used, have $\frac{MP}{Cost}$ be equal, and

$\geq$ to $\frac{MP}{cost}$ of any unused input.

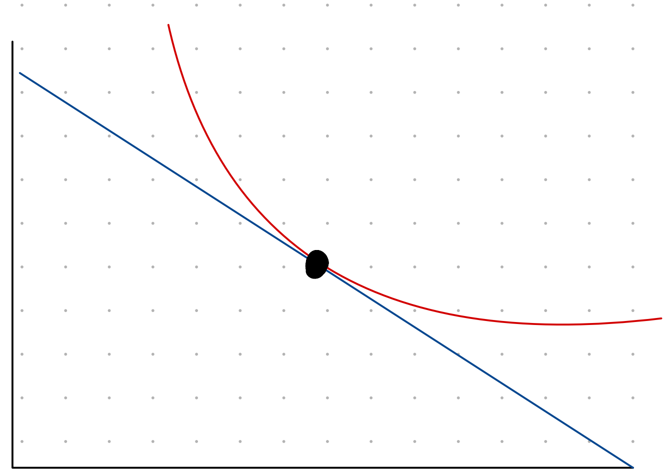
$$\frac{MP_L}{W} = \frac{MP_K}{R}$$

$$\frac{MP_L}{MP_K} = \frac{W}{R}$$

Slope of isocost curve

Marginal rate of Technical substitution

MRTS



Solving a Cost Min. problem

ex: $Q = f(K,L) = K^{.5}L^{.5}$

$C = RK + WL$; $Q^* = 5$

$W = 50{,}000$

$R = 1$

goal: $\min\ RK + WL$

s.t. $K^{.5}L^{.5} = 5$

1. Find MP_L, MP_K (MRTS)

$Q = K^{.5}L^{.5}$

$$MRTS = \frac{MP_L = .5K^{.5}L^{-.5}}{MP_K = .5K^{-.5}L^{.5}} = \frac{K}{L}$$

2. find ratio of prices (slope of isocost line)

$$-\text{Slope} = \frac{W}{R} = 50{,}000$$

3. Set (1) = (2) to ensure $\frac{MP}{\text{Cost}}$ is minimized

note: if this is not possible → corner solution

$$50{,}000 = \frac{K}{L}$$

① $K = 50{,}000L$

4. ~~$C = 1 \cdot (50{,}000L) + 50{,}000L$~~

3 vars
2 eq.

Instead

System of equations with target quantity ①②

② $5 = K^{.5} L^{.5}$

$$5 = (50{,}000L)^{.5} L^{.5}$$

$$5 = \sqrt{50{,}000}\ L$$

$$L = \frac{5}{\sqrt{50{,}000}}$$

① $K = 50{,}000 \cdot \frac{5}{\sqrt{50{,}000}}$

Say $Q = f(K,L) = K^{.25}L^{.75}$, $R=4$, $W=2$.

want to produce 80 units.

What is the solution to the cost minimization problem?

$$\frac{MP_L = .75K^{.25}L^{-.25}}{MP_K = .25K^{-.75}L^{.75}} = \frac{3K}{L} = MRTS_{LK}$$

$$\frac{-W}{R} = \frac{-2}{4} = \text{Slope of isocost curve}$$

$$\frac{-2}{4} = \frac{3K}{L}$$

$L = -6K$ ~~$\times$~~ need slope $\rightarrow$ $\frac{2}{4} = \frac{3K}{L} \rightarrow \underline{\underline{L = 6K}}$

$$80 = K^{.25} L^{.75}$$

$$80 = K^{.25}(6K)^{.75}$$

$$80 = K^{.25} K^{.75} 6^{.75}$$

$$\frac{80}{6^{.75}} = K$$

$K \approx 20.8$ $\qquad$ $L = 6 \cdot 20.8 \approx 125.3$

Special Cobb Douglas Property

$R = 4 \qquad W = 2$

$$83.2 + 250.6 = 330$$

\$ spent on K

~same

\$ spent on L

~same

Total \$ spent on inputs

$$f(K,L) = K^{.25} L^{.75}$$

$$330 \cdot .25 \approx 82.5$$

$$330 \cdot .75 \approx 247.5$$

rounding errors