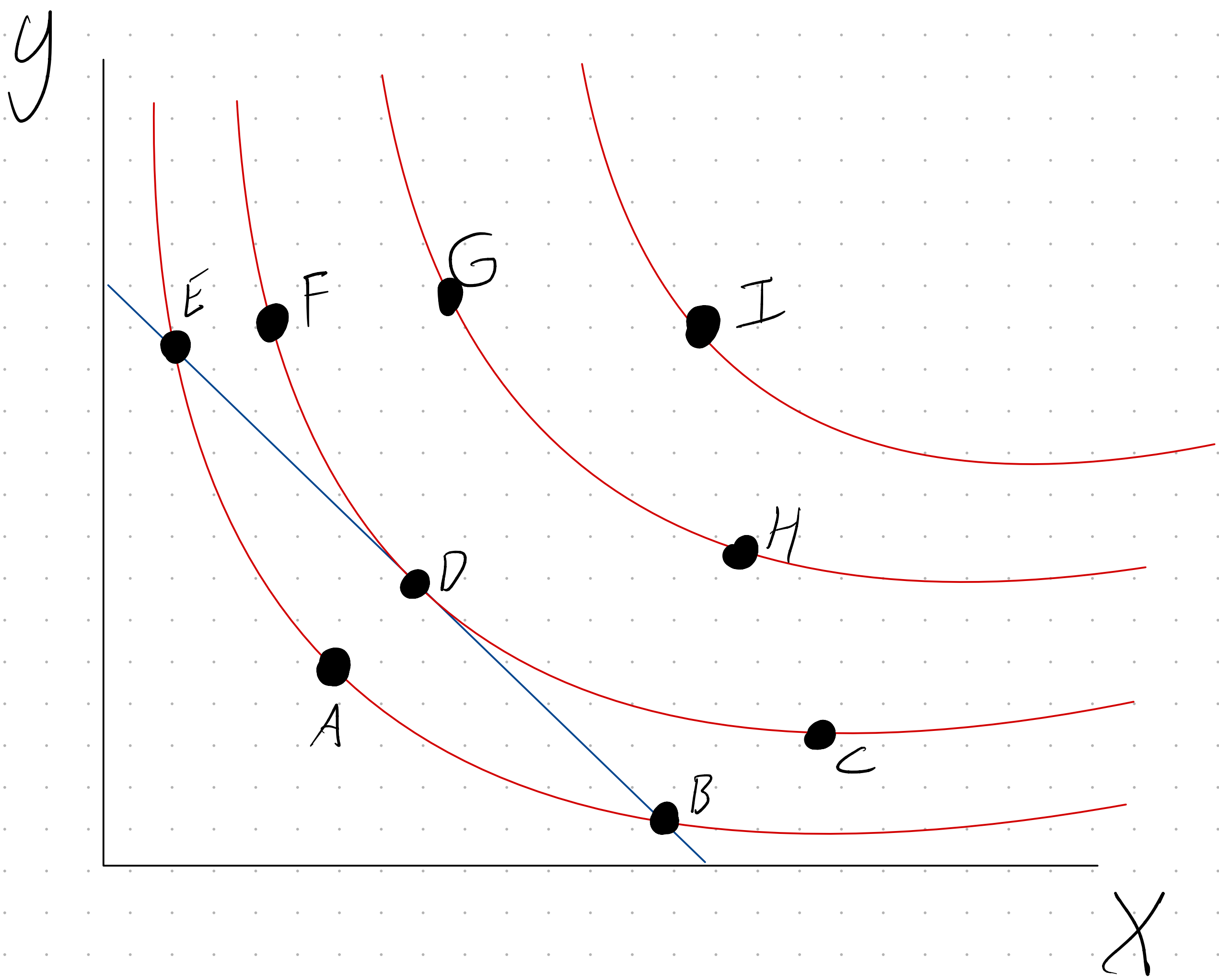
4.4 Combining Utility, income + Prices

Constrained Optimization



Feasible: A, B, D, E

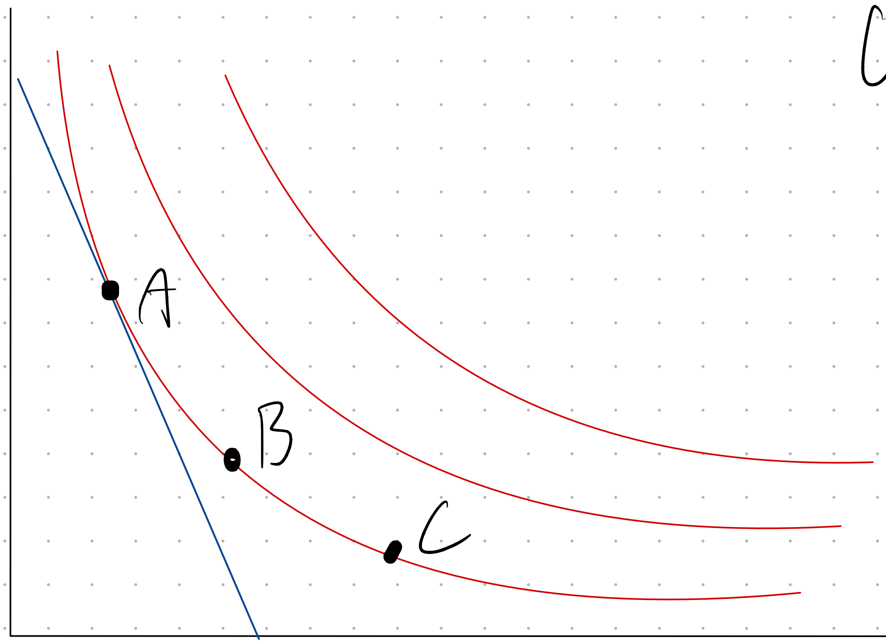
Infeasible: F, C, G, H, I

$$\cancel{I} \succ \cancel{G}, \cancel{H} \succ \cancel{F}, \boxed{D}, \cancel{C} \succ E, A, B$$

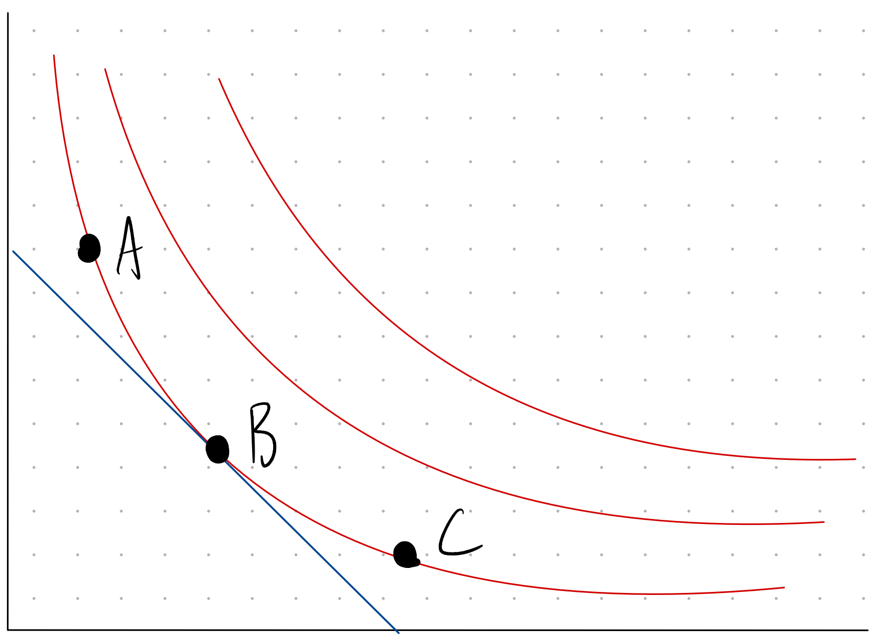
D is the best bundle we can afford.

It is the utility-maximizing bundle subject to our budget constraint.

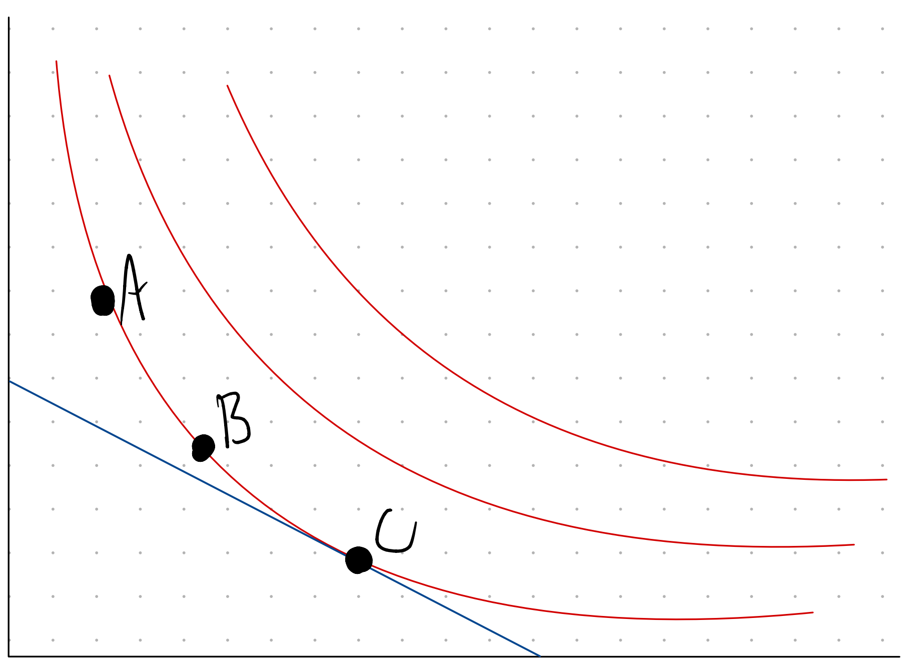
Utility Max. Bundle



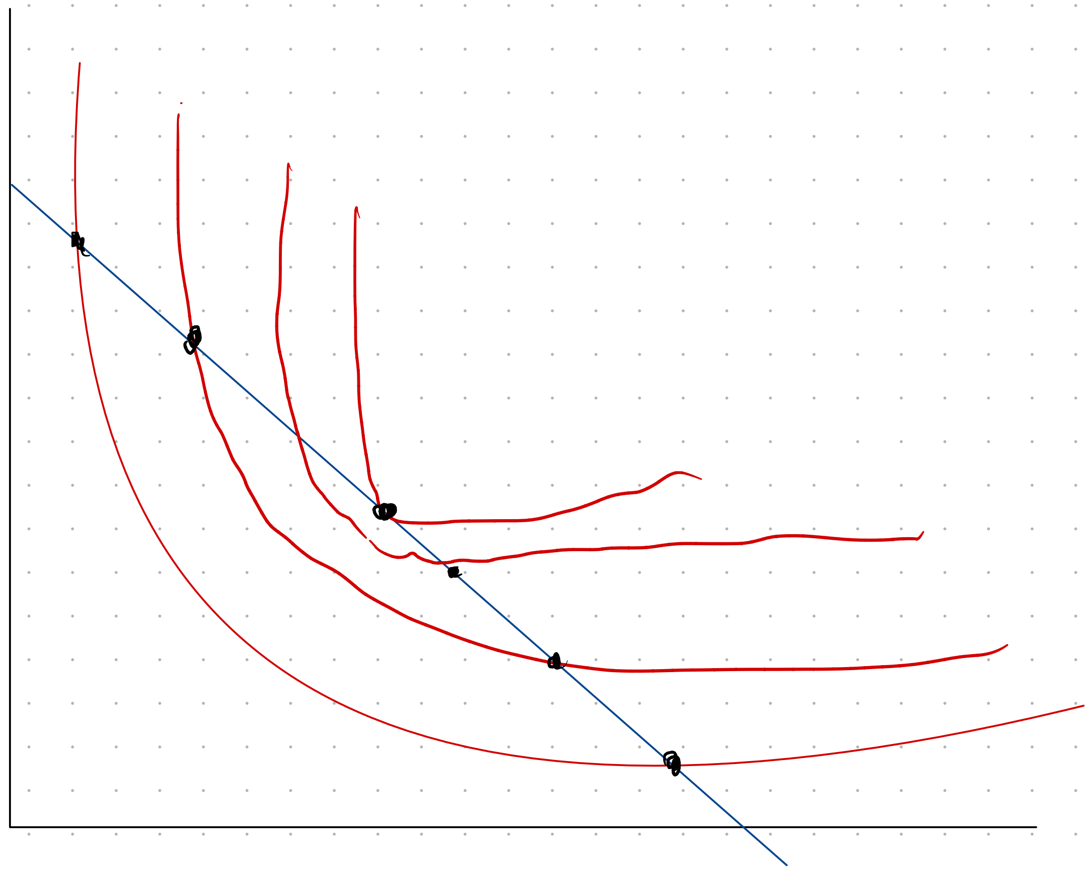
· A



· B



· C



Observation: The budget constraint is tangent to the indifference curve at the utility maximizing bundle.

↳ The slope of the budget constraint is equal to the marginal rate of substitution at the utility max. bundle.

Slope of budget constraint $= -MRS_{xy}$

$$\frac{-P_x}{P_y} = -\frac{MU_x}{MU_y}$$

S.t. Income $=$ cost $= P_x Q_x + P_y Q_y$

this it fixed

$$MU_x P_y = P_x MU_y$$

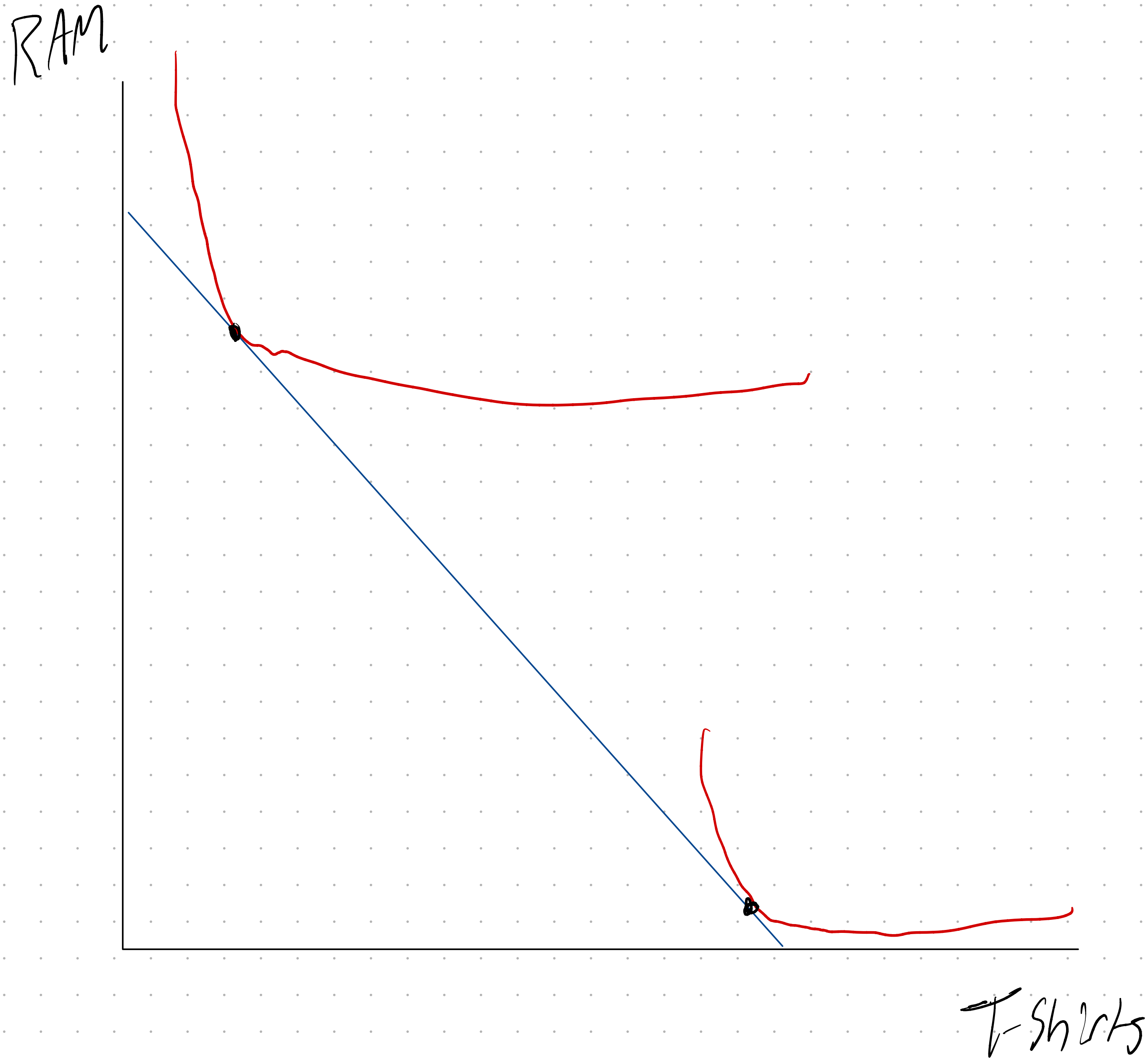
$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

marginal utility per \$ spent is the same for both goods!

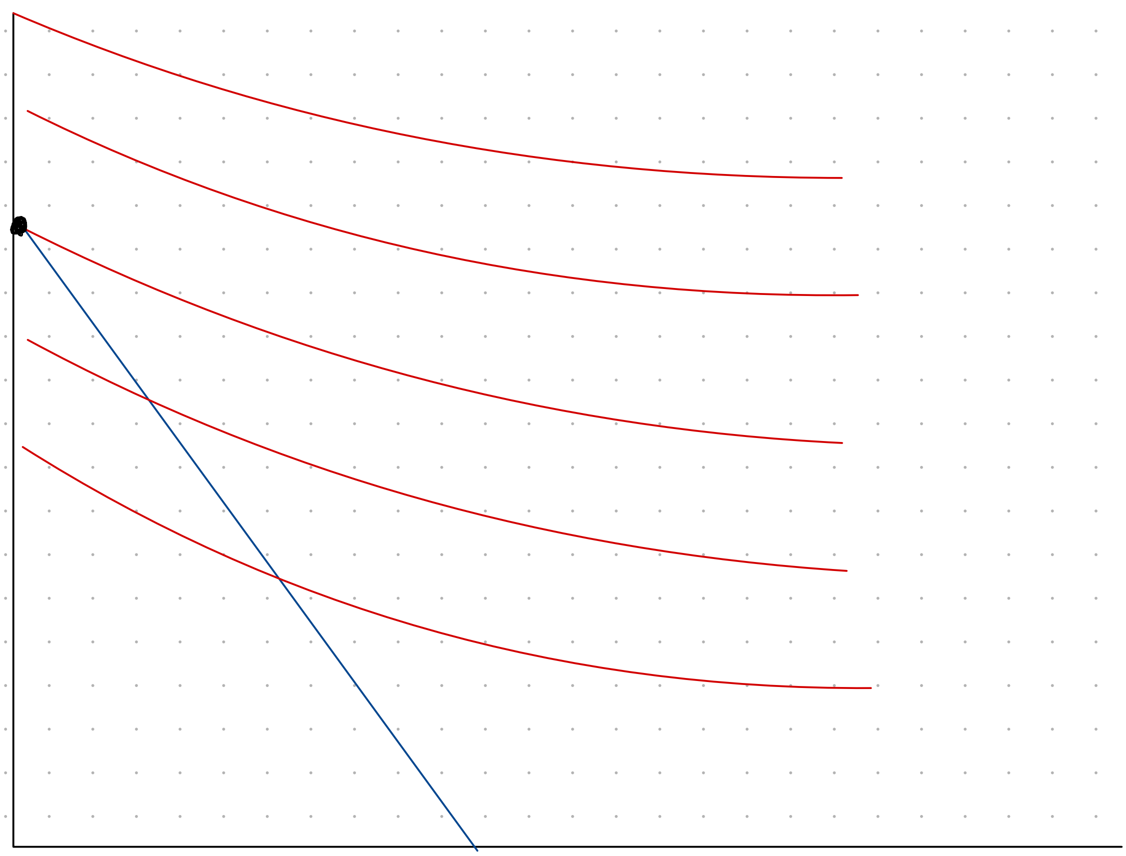
Implications:

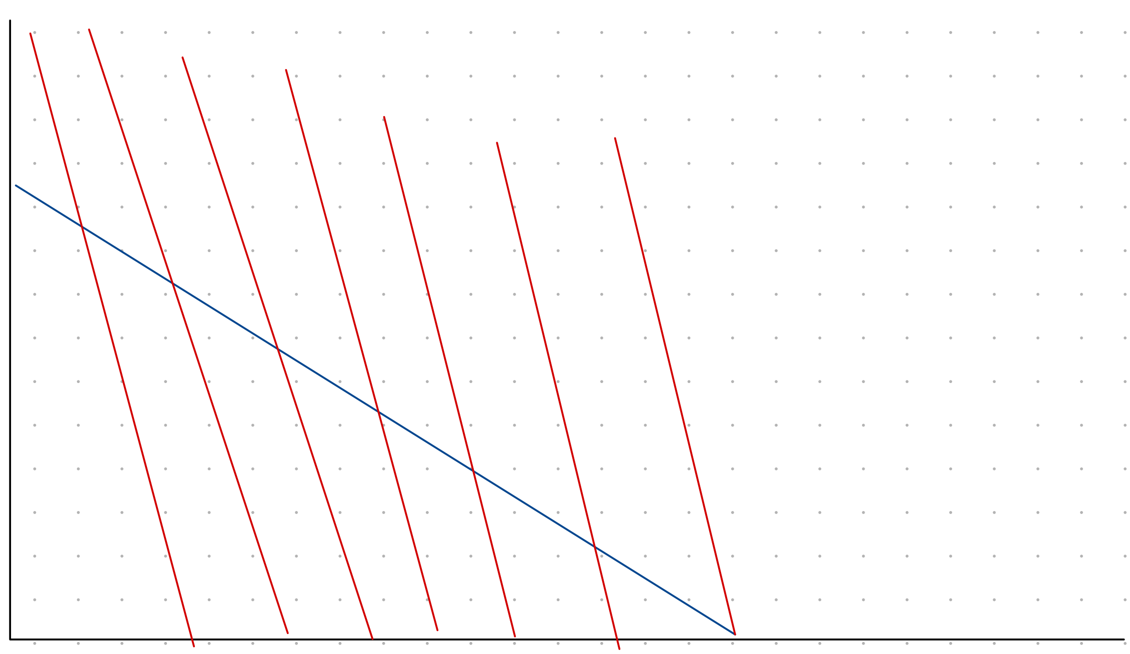
If everyone is utility maximizing and face the same, linear, prices then everyone has the same MRS between the two goods

So long as they are consuming atleast some of each.



- Special cases: Corner solutions



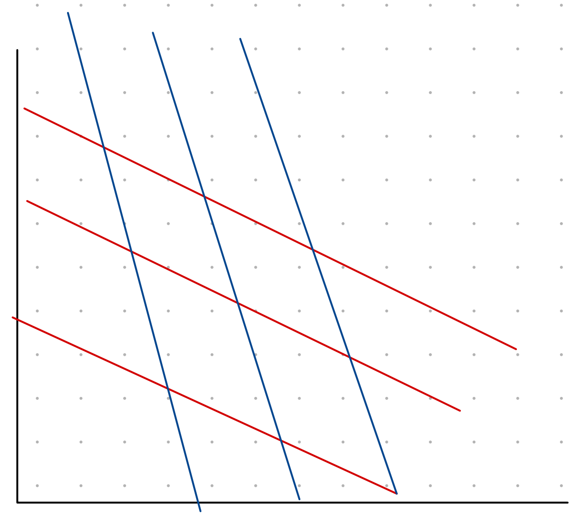


$$U(X,Y) = 5x + 2y$$

Income = \$50

$P_x = \$4$

$P_y = \$1$



$$MRS_{xy} = \frac{-P_x}{P_y}$$

$$\frac{MU_x}{MU_y} = \frac{-4}{1}$$

$$\frac{5}{2} \neq \frac{-4}{1}$$

Spend all \$ on y: 50 → 100

Spend all \$ on x: 12.5 → 62.5

$(0, 50)$

$U(X, y) = X \cdot y$

Income = \$10

$P_x = \$2$

$P_y = \$1$

$$-MRS_{xy} = \frac{-P_x}{P_y}$$

$$\frac{y}{X} = \frac{2}{1}$$

$$\frac{y}{x} = 2 \qquad \cancel{(1,2)}$$

Remember need to spend all our income.

$$\$10 = P_x Q_x + P_y Q_y$$

$$10 = 2 \cdot x + 1 \cdot y$$

$$\frac{y}{x} = 2 \rightarrow y = 2x$$

$$y = 2 \cdot 2.5$$

$$y = 5$$

$$10 = 2x + 2x$$

$$10 = 4x$$

$$2.5 = x$$

$$\boxed{(2.5, 5)}$$

$U(X,Y) = X^{.5}Y^{.5}$

Income = \$100

$P_x = \$5$

$P_y = \$2$

Step 1: $MRS_{xy} = \frac{P_x}{P_y}$

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

$$\frac{.5X^{-.5}Y^{.5}}{.5X^{.5}Y^{-.5}} = \frac{5}{2}$$

$$\frac{Y}{X} = \frac{5}{2}$$

Step 2: Income = Cost

$$\$100 = P_x Q_x + P_y Q_y$$

$$100 = 5X + 2y$$

Step 3: Solve Systems of equations

$$Y = \frac{5}{2}X$$

$$Y = \frac{5}{2} \cdot 10$$

$$Y = 25$$

$$100 = 5X + 2\left(\frac{5}{2}X\right)$$

$$100 = 5x + 5x$$

$$100 = 10x \rightarrow X = 10$$

$$\boxed{(10, 25)}$$